ON SPLITTING OF A BRITTLE BODY BY A WEDGE OF FINITE LENGTH

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This paper deals with the problem of splitting of an infinite isotropic brittle body by an immovable rigid wedge of finite length, symmetrical with respect to its longitudinal axis. The condition of plane strain is assumed, and the friction between the wedge and the surface of the body is neglected. The consideration is carried on within the outlines of the general theory of cracks presented in [1].

1. Formulation of the problem. According to the common concept of splitting, the picture of this process is the following. In the vicinity of the inserted wedge, a crack develops (Fig. 1) whose ends C and D are to be determined in the course of the solution. If the corners of the wedge are rounded, the positions of the terminal points of contact, A and B, are also unknown and should be determined in the solution.

If the wedge is thin enough, the problem may be linearized by reducing the boundary conditions to the line CD, which is assumed as x-axis. The symmetry of the problem allows for dealing with the lower half-plane only (Fig. 2, where a, b, c, d are the coordinates of the points A, B, C, D, respectively). Neglecting the bond forces which exist near the ends of the crack, the boundary conditions for the lower half-plane reduce to the form

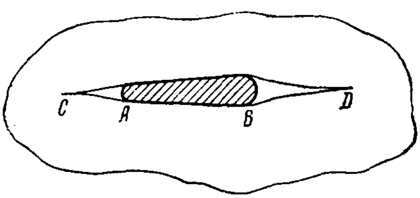

Fig. 1.

$$\frac{\partial v}{\partial x} = 0, \qquad X_y = 0 \qquad \begin{cases} -\infty < x \leqslant c \\ d \leqslant x < \infty \end{cases}$$

$$Y_y = g(x), \qquad X_y = 0 \qquad \begin{cases} c \leqslant x \leqslant a \\ b \leqslant x \leqslant d \end{cases}$$

$$\frac{\partial v}{\partial x} = f'(x), \qquad X_y = 0 \qquad a \leqslant x \leqslant b \qquad (1.1)$$

where $v(x)$ is the component of displacement in the direction of y; Y_y and X_y are the components of stress tensor; $f(x)$ is the function determining the shape of the wedge in a coordinate system connected with the wedge; $-g(x)$ is the tensile stress in the continuous uncracked body on the surfaces of free cracks caused by external loading acting on the body with the wedge.

According to the method of Muskhelishvili [2], the components of the stress and deformation tensors can be expressed in terms of one analytic function $\Phi(z)$:

$$
\begin{aligned}
&X_x + Y_y = 4\,\mathrm{Re}\,\Phi(z) \\
&Y_y - X_x + 2iX_y = 2\left[(\bar{z} - z)\,\Phi'(z) - \Phi(z) - \overline{\Phi}(\bar{z})\right] \\
&2\mu\,(u' + iv') = \varkappa\Phi(z) + \Phi(\bar{z}) - (z - \bar{z})\,\overline{\Phi'(z)}
\end{aligned}
\tag{1.2}
$$

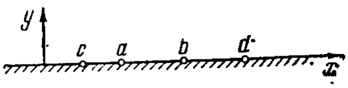

Fig. 2.

If, on the boundary of the half-plane $X_y = 0$, then, on this boundary

$$
Y_y = 2\,\mathrm{Re}\,\Phi(z), \qquad \frac{\partial v}{\partial x} = \frac{\varkappa + 1}{2\mu}\,\mathrm{Im}\,\Phi(z) \tag{1.3}
$$

i.e.

$$
\mathrm{Re}\,\Phi(z) = \frac{1}{2}\,Y_y, \qquad \mathrm{Im}\,\Phi(z) = \frac{2\mu}{\varkappa + 1}\,\frac{\partial v}{\partial x}
$$

To determine the function $\Phi(z)$ in the lower half-plane, we obtain the following mixed boundary-value problem:

$$
\begin{aligned}
&\mathrm{Im}\,\Phi(z) = 0 && (-\infty < x \leqslant c, \quad d \leqslant x < \infty) \\
&\mathrm{Re}\,\Phi(z) = \frac{1}{2}\,g(x) \quad \begin{cases} c \leqslant x \leqslant a, \\ b \leqslant x \leqslant d, \end{cases} && \mathrm{Im}\,\Phi(z) = \frac{2\mu}{\varkappa + 1}\,f'(x) \qquad a \leqslant x \leqslant b
\end{aligned}
\tag{1.4}
$$

2. Solution of the boundary-value problem and determination of the constants of the solution. The boundary-value problem obtained can be solved by the use of the formula of Keldysh and Sedov [3] in the form

$$
\Phi(z) = \frac{1}{\pi i X(z)}\left\{\frac{1}{2}\int_c^a g(t)\,X(t)\,\frac{dt}{t - z} + \frac{1}{2}\int_b^d g(t)\,X(t)\,\frac{dt}{t - z} - \right. \tag{2.1}
$$

$$
\left. - i\,\frac{2\mu}{\varkappa + 1}\int_a^b f'(t)\,X(t)\,\frac{dt}{t - z}\right\} + \frac{C_0 + C_1 z}{X(z)} \qquad \left(X(z) = \sqrt{(z - c)(z - a)(z - b)(z - d)}\right)
$$

The real constants C_0 and C_1 have to be determined together with the constants c, a, b, d, which appear in Expression (2.1).

We find the displacement $v(x)$ of the points of the crack beyond the region of contact with the wedge. Substituting (2.1) into (1.3) and integrating, we obtain for $c \leqslant x \leqslant a$

$$v(x) = \frac{\varkappa+1}{4\pi\mu}\int_c^x \frac{1}{X_2(x)}\left[\int_c^a g(t)\,X_2(t)\,\frac{dt}{t-x} - \int_b^d g(t)\,X_4(t)\,\frac{dt}{t-x}\right]dx - \tag{2.2}$$

$$-\frac{1}{\pi}\int_c^x \frac{1}{X_2(x)}\left[\int_a^b f'(t)\,X_3(t)\,\frac{dt}{t-x}\right]dx - \frac{(\varkappa+1)\,C_0}{2\mu}\int_c^x \frac{dt}{X_2(t)} - \frac{(\varkappa+1)\,C_1}{2\mu}\int_c^x \frac{tdt}{X_2(t)} + D_2$$

$$\left(\begin{array}{c} X_2(z) = \sqrt{(z-c)(a-z)(b-z)(d-z)}, \quad X_3(z) = \sqrt{(z-c)(z-a)(b-z)(d-z)} \\ X_4(z) = \sqrt{(z-c)(z-a)(z-b)(d-z)} \end{array}\right)$$

with the integration constant D_1 equal to zero, because at the end of the crack (for $x = c$) the displacement is equal to zero. At the point of contact A, the displacement $v(x)$ is equal to the given value $f(a)$, and therefore

$$f(a) = \frac{\varkappa+1}{4\pi\mu}\int_c^a \frac{1}{X_2(x)}\left[\int_c^a g(t)\,X_2(t)\,\frac{dt}{t-x} - \int_b^d g(t)\,X_4(t)\,\frac{dt}{t-x}\right]dx -$$

$$-\frac{1}{\pi}\int_c^a \frac{1}{X_2(x)}\left[\int_a^b f'(t)\,X_3(t)\,\frac{dt}{t-x}\right]dx - \frac{\varkappa+1}{2\mu}\,C_0 J_1 - \frac{\varkappa+1}{2\mu}\,C_1 J_2 \tag{2.3}$$

Here

$$J_1 = \int_c^a \frac{dt}{X_2(t)} = \frac{4}{m_1+m_2}\,F\left(\frac{m_1-m_2}{m_1+m_2}\right), \qquad \begin{array}{l} n_1 = \sqrt{(b-c)(d-c)}, \quad n_2 = \sqrt{(b-a)(d-a)} \\ m_1 = \sqrt{(b-c)(d-a)}, \quad m_2 = \sqrt{(b-a)(d-c)} \end{array}$$

$$J_2 = \int_c^a \frac{tdt}{X_2(t)} = \frac{an_1 - cn_2}{n_1-n_2}J_1 - \frac{8n_1n_2}{(n_1^2-n_2^2)(m_1+m_2)}\,H\left[-\left(\frac{n_1-n_2}{n_1+n_2}\right)^2;\ \frac{m_1-m_2}{m_1+m_2}\right]$$

where F and H are the usual symbols for complete elliptic integrals of the first and second kind, respectively. We obtain, similarly, the second condition

$$f(b) = \frac{\varkappa+1}{\pi\mu}\int_b^d \frac{1}{X_4(x)}\left[\int_c^a g(t)\,X_2(t)\,\frac{dt}{t-x} - \int_b^d g(t)\,X_4(t)\,\frac{dt}{t-x}\right]dx -$$

$$-\frac{1}{\pi}\int_b^d \frac{1}{X_4(x)}\left[\int_a^b f'(t)\,X_3(t)\,\frac{dt}{t-x}\right]dx - \frac{\varkappa+1}{2\mu}\,C_0 J_3 - \frac{\varkappa+1}{2\mu}\,C_1 J_4 \tag{2.4}$$

Here

$$J_3 = \int_b^d \frac{dt}{X_4(t)} = J_1 \qquad l_1 = \sqrt{(b-c)(b-a)}, \qquad l_2 = \sqrt{(d-c)(d-a)}$$

$$J_4 = \int_b^d \frac{tdt}{X_4(t)} = \frac{dl_1 - bl_2}{l_1 - l_2} J_1 - \frac{8 l_1 l_2}{(l_1^2 - l_2^2)(m_1 + m_2)} \Pi \left[-\left(\frac{l_1 - l_2}{l_1 + l_2}\right)^2;\ \frac{m_1 - m_2}{m_1 + m_2} \right]$$

We find now the distribution of stresses in the area of contact of the wedge and the half-plane. Substituting (2.1) into the first of Equations (1.3), we obtain

$$Y_y = \frac{1}{\pi X_3(x)} \left\{ \int_c^a g(t) X_2(t) \frac{dt}{t-x} - \int_b^d g(t) X_4(t) \frac{dt}{t-x} - \right.$$

$$\left. - \frac{2\mu}{\varkappa + 1} \int_a^b f'(t) X_3(t) \frac{dt}{t-x} - 2\pi C_0 - 2\pi C_1 x \right\} \qquad (a \leqslant x \leqslant b) \qquad (2.5)$$

At the points A and B (Fig. 1), i.e. for $x = a$ and $x = b$, the stresses should be bounded (for the case of rounded corners). Hence, we have

$$\int_c^a \frac{g(t) X_2(t) dt}{t-a} - \int_b^d \frac{g(t) X_4(t) dt}{t-a} - \frac{2\mu}{\varkappa+1} \int_a^b \frac{f'(t) X_3(t) dt}{t-a} - 2\pi (C_0 - C_1 a) = 0 \qquad (2.6)$$

$$\int_c^a \frac{g(t) X_2(t) dt}{t-b} - \int_b^d \frac{g(t) X_4(t) dt}{t-b} - \frac{2\mu}{\varkappa+1} \int_a^b \frac{f'(t) X_3(t) dt}{t-b} - 2\pi (C_0 - C_1 b) = 0 \qquad (2.7)$$

In order to determine the positions of the ends of the crack, we require that the tensile stress in the vicinity of these ends, calculated without considering the bond forces, be of the order $K/\pi \sqrt{s}$, where K is the modulus of cohesion and s is the distance from the end of the crack. Using this condition, we note that the stresses beyond the crack (for $-\infty < x \leqslant c$, $d \leqslant x < \infty$) are

$$Y_y = \frac{1}{\pi X(x)} \left\{ - \int_c^a g(t) X_2(t) \frac{dt}{t-x} + \int_b^d g(t) X_4(t) \frac{dt}{t-x} + \right.$$

$$\left. + \frac{2\mu}{\varkappa+1} \int_a^b f'(t) X_3(t) \frac{dt}{t-x} + 2\pi C_0 + 2\pi C_1 x \right\} \qquad (2.8)$$

Hence

$$-\int_c^a g(t)\,X_2(t)\,\frac{dt}{t-c}+\int_b^d g(t)\,X_4(t)\,\frac{dt}{t-c}+\frac{2\mu}{\varkappa+1}\int_a^b f'(t)\,X_3(t)\,\frac{dt}{t-c}+$$

$$+2\pi C_0+2\pi C_1 c=K\sqrt{(a-c)(b-c)(d-c)} \tag{2.9}$$

$$-\int_c^a g(t)\,X_2(t)\,\frac{dt}{t-d}+\int_b^d g(t)\,X_4(t)\,\frac{dt}{t-d}+\frac{2\mu}{\varkappa+1}\int_a^b f'(t)\,X_3(t)\,\frac{dt}{t-d}+$$

$$+2\pi C_0+2\pi C_1 d=K\sqrt{(d-c)(d-a)(d-b)} \tag{2.10}$$

In this way, we have six relations, (2.3) to (2.4), (2.6) to (2.7), and (2.9) to (2.10), for the six sought constants.

We consider the case of a wedge symmetrical with respect to the axis y, i.e. $a=-b$, $c=-d=-l$, and we obtain

$$\Phi(z)=\frac{1}{\pi i\,X(z)}\left[\frac{1}{2}\int_{-l}^{-b} g(t)\,X(t)\,\frac{dt}{t-z}+\frac{1}{2}\int_b^l g(t)\,X(t)\,\frac{dt}{t-z}-\right. \tag{2.11}$$

$$\left.-i\,\frac{2\mu}{\varkappa+1}\int_{-b}^{b} f'(t)\,X(t)\,\frac{dt}{t-z}\right]+\frac{C_0+C_1 z}{X(z)} \qquad \left(X(z)=\sqrt{(z^2-b^2)(z^2-l^2)}\right)$$

Because the functions $f(x)$ and $g(x)$ are even, it is easy to show that $C_1=0$. The six relations between the other constants can be reduced to the following three equations:

$$f(b)=\frac{\varkappa+1}{4\pi\mu}\int_b^l \frac{1}{\sqrt{(l^2-x^2)(x^2-b^2)}}\left[\int_{-l}^{-b} g(t)\sqrt{(l^2-t^2)(t^2-b^2)}\,\frac{dt}{t-x}-\right.$$

$$\left.-\int_b^l g(t)\sqrt{(l^2-t^2)(t^2-b^2)}\,\frac{dt}{t-x}\right]dx-$$

$$-\frac{1}{\pi}\int_b^l \frac{1}{\sqrt{(l^2-x^2)(x^2-b^2)}}\left[\int_{-b}^{b} f'(t)\sqrt{(l^2-t^2)(b^2-t^2)}\,\frac{dt}{t-x}\right]dx-\frac{\varkappa+1}{2\mu}J_0 C_0 \tag{2.12}$$

$$\int_{-l}^{-b} g(t)\sqrt{l^2-t^2}\sqrt{\frac{t+b}{t-b}}\,dt-\int_b^l g(t)\sqrt{l^2-t^2}\sqrt{\frac{t+b}{t-b}}\,dt+$$

$$+\frac{2\mu}{\varkappa+1}\int_{-b}^{b} f'(t)\sqrt{l^2-t^2}\sqrt{\frac{b+t}{b-t}}\,dt-2\pi C_0=0 \tag{2.13}$$

$$\int_{-l}^{-b} g(t)\sqrt{t^2-b^2}\sqrt{\frac{l+t}{l-t}}\,dt-\int_b^l g(t)\sqrt{t^2-b^2}\sqrt{\frac{l+t}{l-t}}\,dt-$$

$$-\frac{2\mu}{\varkappa+1}\int_{-b}^{b} f'(t)\sqrt{b^2-t^2}\sqrt{\frac{l+t}{l-t}}\,dt+2\pi C_0=K\sqrt{2l}\sqrt{l^2-b^2} \qquad (2.14)$$

where

$$J_0=\int_{b}^{l}\frac{dx}{\sqrt{(l^2-x^2)(x^2-b^2)}}=\frac{1}{l}F(k), \qquad k=\frac{\sqrt{l^2-b^2}}{l}$$

and $F(k)$ is the complete elliptic integral of the first kind with the modul k.

3. Solution of the problem of splitting by a strip of finite width in a stationary field. Consider an infinite plate, with Young's modulus E, Poisson's ratio ν, and the cohesion modulus K, in a homogeneous stress field Q. Let a rigid wedge of thickness $2h$ and width $2b$ be inserted into

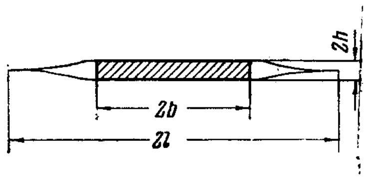

Fig. 3.

this plate. It produces a crack from $-l$ to $+l$. The coordinates of the points of contact of the crack and the wedge are known ($x_1=-b$, $x_2=b$) in advance, but the stresses at these points are infinite.

Using Equation (2.11) and considering that $f'(x)=0$, $g(x)=-Q$ we have

$$\Phi(z)=-\frac{Q}{2}+\frac{Qz^2}{2\sqrt{(z^2-l^2)(z^2-b^2)}}+\frac{C_0'}{2\sqrt{(z^2-l^2)(z^2-b^2)}} \qquad (C_0'=2C_0). \qquad (3.1)$$

The distribution of displacements on the segment $b\leqslant x\leqslant l$ is of the form

$$v(x)=\frac{\varkappa+1}{4\mu}\left[\int_{b}^{x}\frac{C_0'\,dx}{\sqrt{(l^2-x^2)(x^2-b^2)}}+\int_{b}^{x}\frac{Qx^2\,dx}{\sqrt{(x^2-b^2)(l^2-x^2)}}\right]-h$$

(with $v(b)=-h$). From the fact that $v(l)=0$, we obtain

$$C_0'\int_{b}^{l}\frac{dx}{\sqrt{(l^2-x^2)(x^2-b^2)}}+Q\int_{b}^{l}\frac{x^2\,dx}{\sqrt{(l^2-x^2)(x^2-b^2)}}-\frac{Eh}{2(1-\nu^2)}=0$$

Noting that

$$\int_b^l \frac{dx}{\sqrt{(l^2-x^2)(x^2-b^2)}} = \frac{F(k)}{l}, \qquad \int_b^c \frac{x^2\,dx}{\sqrt{(l^2-x^2)(x^2-b^2)}} = lE(k)$$

where $E(k)$ is the complete elliptic integral of the second kind with modulus k, introduced at the end of Section 2, we obtain

$$C_0' = -Ql^2\frac{E(k)}{F(k)} + \frac{Ehl}{2(1-\nu^2)F(k)} \tag{3.2}$$

In order to determine the length of the crack, we use the relation which determines the length of an equilibrium crack in an infinite body [1]

$$\int_0^l \frac{p(x)\,dx}{\sqrt{l^2-x^2}} = \frac{K}{\sqrt{2l}} \tag{3.3}$$

where $p(x)$ is the distribution of pressure on the surface of the crack. For the points under the wedge

$$p(x) = \frac{Qx^2 + C_0'}{\sqrt{(l^2-x^2)(b^2-x^2)}} + Q \qquad (-b \leqslant x \leqslant b) \tag{3.4}$$

Considering, in addition, that at the points of the free crack $(b \leqslant x \leqslant l)$ the pressure Q is applied, and using (3.3), we have

$$Q\int_0^b \frac{x^2\,dx}{\sqrt{b^2-x^2}\,(l^2-x^2)} + Q\int_0^l \frac{dx}{\sqrt{l^2-x^2}} + C_0'\int_0^b \frac{dx}{\sqrt{b^2-x^2}\,(l^2-x^2)} = \frac{K}{\sqrt{2l}}$$

Hence, evaluating the integrals in this expression, we obtain the relation determining the length of the equilibrium crack:

$$(C_0' + Ql^2)\,\pi = K\sqrt{2l}\sqrt{l^2-b^2} \tag{3.5}$$

It is understandable that the relation (3.5) has been obtained from the condition that the tensile stress Y_y in the vicinity of $x = l$, calculated without considering the bond forces, is of the order $K/\pi\sqrt{(x-l)}$.

Substituting the value of C_0' given by (3.2) into (3.5), we represent the condition determining the length of the crack in the form

$$\frac{Ql}{\sqrt{l^2-b^2}}\left[\frac{F(k)-E(k)}{F(k)}\right] + \frac{Eh}{2(1-\nu^2)F(k)\sqrt{l^2-b^2}} = \frac{K\sqrt{2}}{\pi\sqrt{l}} \tag{3.6}$$

This fundamental relation can be written in the shorter form

$$\lambda_1 A(k) + B(k) = \mu_1$$

where

$$A(k) = \frac{F(k) - E(k)}{kF(k)\sqrt[4]{1-k^2}}, \quad B(k) = \frac{\sqrt[4]{1-k^2}}{kF(k)}$$

$$\lambda_1 = \frac{2Qb(1-\nu^2)}{Eh}, \quad \mu_1 = \frac{2\sqrt{2}K(1-\nu^2)\sqrt{b}}{\pi Eh}$$

Figure 4 shows the diagrams which facilitate the calculations of the length of the crack for any given Q, b and h.

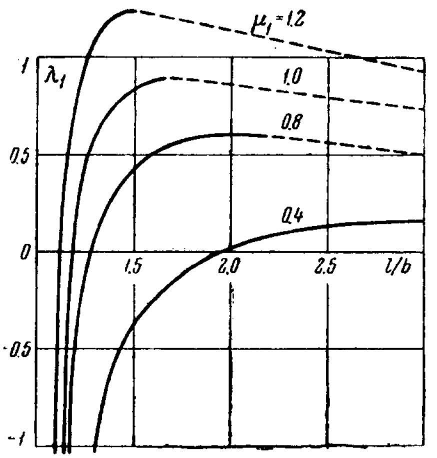

Fig. 4.

If the external loading produces tensile stresses ($\lambda_1 > 0$), the diagram of the function $(\mu_1 - B)/A$ has two branches: a stable and an unstable, with respect to Q. The branches corresponding to the unstable cracks are indicated by broken lines. For increasing tensile stress, the length of the crack initially increases, but the crack remains stable. If, however, the value of Q exceeds a certain critical value Q^*, which depends on the properties of the material and the parameters of the wedge, then there is no solution to the considered problem. In the case of a compressive stress, only stable cracks exist. Substituting into (3.1), (3.5) and (3.6), we obtain the solution of the problem of splitting of an infinite plate by a strip of finite width without external loading. The relation which determines the length of the crack then assumes the form

$$\frac{\sqrt[4]{1-k^2}}{kF(k)} = \frac{2\sqrt{2}K(1-\nu^2)\sqrt{b}}{E\pi h} \qquad (B(k) = \mu_1) \qquad (3.7)$$

Since the function $B(k)$ is monotonically decreasing, to each value of h corresponds only one, uniquely determined, length of the crack. This length increases with increasing h.

The relation (3.7) can be written in the form

$$\frac{l-b}{L_0} = \frac{\pi^2}{2} B^2(k) \left(\frac{b}{L_0}\right) \tag{3.8}$$

where L_0 is the length of the free crack developed at the end of a semi-infinite wedge [1]. Thus, the ratio of the length of the crack at the ends of a wedge of finite width and the length of the crack at the end of a semi-infinite wedge does not depend on h, E, k, ν. Figure 5 shows the relation between $(l - b)/L_0$ and b/L_0.

From the relations (3.4) and (3.6), the solution can be obtained for the problem of a semi-infinite wedge splitting a plate in a homogeneous stress field. The distribution of stresses on the surfaces of the wedge can be obtained as the limit of Expression (3.4). Substituting $x = x_1 - l$ and assuming that l and b increase to infinity in such a way that the difference $l - b = L$ remains constant, we obtain

$$p_1(x_1) = \left(\frac{Eh}{2\pi(1-\nu^2)} + \frac{QL}{2} - Qx_1\right) \frac{1}{\sqrt{x_1}\sqrt{x_1 - L}} + Q \tag{3.9}$$

The quantity L in Expression (3.9) can be determined also by a similar limiting process in (3.6). We have

$$Q\sqrt{L} + \frac{Eh}{(1-\nu^2)\pi\sqrt{L}} = \frac{2K}{\pi} \tag{3.10}$$

This relation gives an explicit expression for the length of the crack L as a function of applied loading. We introduce the parameter α defined by

$$Q = \frac{(1-\alpha^2)m^2}{4n} \qquad (-1 < \alpha < \infty),$$

$$\frac{2K}{\pi} = m, \qquad \frac{Eh}{\pi(1-\nu^2)} = n$$

Then

$$\sqrt{L} = \frac{2n}{m}\frac{1}{\alpha+1} \tag{3.11}$$

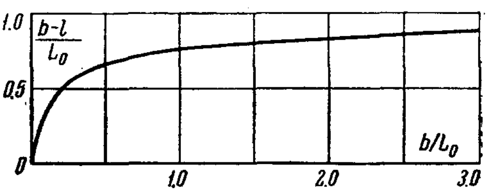

Fig. 5.

For $Q = 0$ $(\alpha = 1)$ we obtain

$$L_0 = \frac{n^2}{m^2} = \frac{E^2h^2}{4(1-\nu^2)^2K^2}$$

This result fully coincides with the result obtained in [1]. In the case of tensile stresses, two branches exist, as in the problem of the wedge of finite width. In the stable region $0 < \alpha < 1$, the length of the crack increases with increasing Q. In the unstable region $-1 < \alpha < 0$, the length of the crack decreases with increasing Q.

If $Q > Q^* = m^2/4n$, then the solution becomes meaningless. The maximum length of the crack cannot exceed $4L_0$. In the case of compressive stresses, the length of the crack decreases with increasing Q, and it is always smaller than L_0.

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